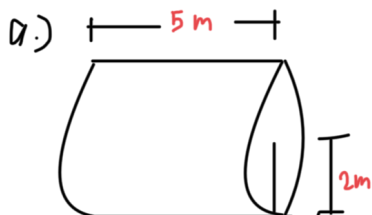


Name: YLARDE, KETHLIR JHON S.

Course & Year: BSCEN-3

1. You are tasked to design a cylindrical water tank for a small community. The tank with radius $r = 2\text{ m}$ and length $L = 5\text{ m}$ must hold a maximum volume of 30 cubic meters.
- What is the maximum height of water that can be stored if the tank is horizontal? (5 points)
 - If a vertical tank of the same radius is used instead while maintaining the same air space between the water level and tank in the horizontal position, what will be the height of the tank? (5 points)
 - Which tank orientation is more economical and why? (5 points)



$$V_{\max} = 30\text{ m}^3$$

$$\begin{aligned} x^2 + y^2 &= r^2 \\ x_0^2 &= r^2 - (h-r)^2 \\ x_0 &= \sqrt{r^2 - (h-r)^2} \\ x_0 &= \sqrt{2hr - h^2} \end{aligned}$$

$$\begin{aligned} A(h) &= \text{sector area} - \text{triangle area} \\ &= \frac{1}{2} r^2 \theta - \frac{1}{2} (2x_0)(r-h) \\ &= \frac{1}{2} r^2 \theta - (r-h) \sqrt{2hr - h^2} \end{aligned}$$

$$V = A(h)L \rightarrow A(h) = \frac{V}{L} = \frac{30\text{ m}^3}{5\text{ m}} = 6.0\text{ m}^2$$

Since 30 m^3 is $V_{\max}$, then $A(h_{\max}) = 6.0\text{ m}^2$

We need to find the root, that is the " h ", by using bisection Method.

$$f(h) = A(h) - 6.0$$

$$f(h) = r^2 \cos^{-1} \left(1 - \frac{h}{r} \right) - (r-h) \sqrt{2hr - h^2} - 6.0 \quad \text{where } 0 \leq h \leq 4$$

$$h_1 = 1.5\text{ m} \quad f(h_1) = -1.6958$$

$$h_2 = 2\text{ m} \quad f(h_2) = 0.2832$$

$$f(h_1)f(h_2) < 0, \text{ at least one root}$$

Using Bisection Method,

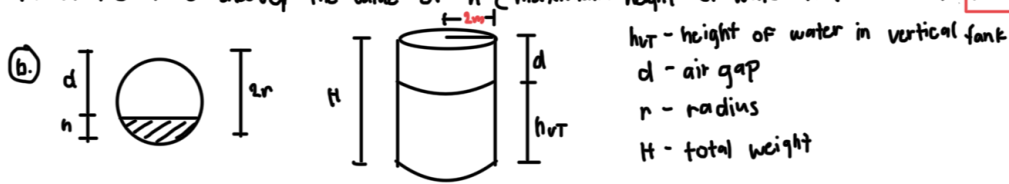
$$h_3 = \frac{h_1 + h_2}{2}$$

tolerance is
 $\leq 1\%$

i	h_1	h_2	h_3	$f(h_1)$	$f(h_2)$	$f(h_3)$	%error
0	1.5	2	1.75	-1.6958	0.2832	-0.7142	
1	1.75	2	1.875	-0.7142	0.2832	-0.2165	6.67%
2	1.875	2	1.9375	-0.2165	0.2832	0.0332	3.23%
3	1.875	1.9375	1.9063	-0.2165	0.0332	-0.0915	1.64%
4	1.9063	1.9375	1.9219	-0.0915	0.0332	-0.0291	0.81%
5	1.9219	1.9375	1.9297	-0.0291	0.0332	0.0020	0.40%

$$\% \text{ error} = \frac{\text{present value} - \text{previous value}}{\text{present value}} (100)$$

From the table above, the value of h (maximum height of water is 1.9297m . $\rightarrow h = 1.9297\text{m}$)



$$d = 2r - h = 2(2) - 1.9297 = 2.0703\text{m}$$

$$h_{vr} = H - d$$

$$V_{\text{water}} = \pi r^2 h_{vr}$$

$$V_{\text{water}} = \pi r^2 (H - d)$$

$$30 = \pi (2)^2 (H - 2.0703)$$

$$H = \frac{30}{4\pi} + 2.0703$$

$$H = 4.4576\text{m}$$

(c.) Both tanks have two circular ends of equal area $2\pi r^2 = 8\pi$ (same radius), so the difference will be on their lateral area (total area of the sides of a 3D object).

Horizontal Tank:

$$\begin{aligned}
 LA_{HT} &= 2\pi rL \\
 &= 2\pi(2)(5) \\
 &= 20\pi \text{ or } 62.8319\text{m}^2
 \end{aligned}$$

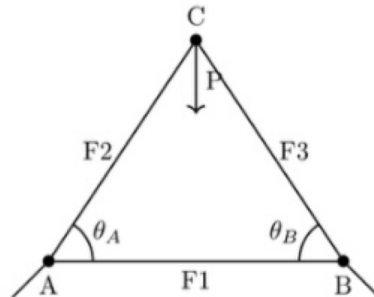
Vertical Tank

$$\begin{aligned}
 LA_{VT} &= 2\pi rH \\
 &= 2\pi(2)(4.4576) \\
 &= \frac{111.44}{6.25}\pi \text{ or } 56.0189\text{m}^2
 \end{aligned}$$

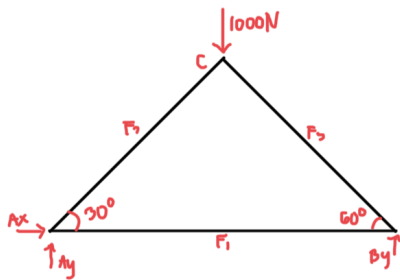
From the above comparison, we can say that the vertical tank is more economical than the horizontal tank because it has smaller lateral area meaning it's more beneficial in terms of shell material / material cost.

2. An important problem in structural engineering is that of finding the forces in a statically determinate truss. This type of structure can be described as a system of coupled linear algebraic equations derived from force balances. The sum of the forces in both horizontal and vertical directions must be zero at each node, because the system is at rest.

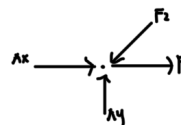
Consider the following truss structure:



The truss is supported at nodes A (hinge) and B (roller), and a downward load $P = 1000 \text{ N}$ is applied at node C. Express the set of equilibrium equations for the truss in matrix form $[K]\{F\} = \{P\}$, where $\{F\}$ is the vector of member forces and reactions, and $\{P\}$ is the load vector. With $\theta_A = 30^\circ$ and $\theta_B = 60^\circ$, determine the forces in members F1, F2, and F3 and the reactions. (20 points)



@ Node A



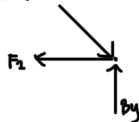
$$\sum F_x = 0$$

$$Ax + F_1 - F_2 \cos 30^\circ = 0$$

$$\sum F_y = 0$$

$$Ay - F_2 \sin 30^\circ = 0$$

@ Node B



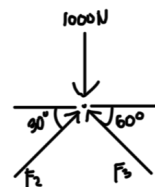
$$\sum F_x = 0$$

$$-F_1 + F_3 \cos 60^\circ = 0$$

$$\sum F_y = 0$$

$$By - F_3 \sin 60^\circ = 0$$

@ Node C



$$\sum F_x = 0$$

$$F_2 \cos 30^\circ - F_3 \cos 60^\circ = 0$$

$$\sum F_y = 0$$

$$F_2 \sin 30^\circ + F_3 \sin 60^\circ - 1000 = 0$$

Unknowns are $F_1, F_2, F_3, A_x, A_y,$ and B_y .

$$\{F\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{bmatrix}$$

$$\{P\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 1 & -\cos 30 & 0 & 1 & 0 & 0 \\ 0 & -\sin 30 & 0 & 0 & 1 & 0 \\ -1 & 0 & \cos 60 & 0 & 0 & 0 \\ 0 & 0 & -\sin 60 & 0 & 0 & 1 \\ 0 & \cos 30 & -\cos 60 & 0 & 0 & 0 \\ 0 & \sin 30 & \sin 60 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\cos 30 & 0 & 1 & 0 & 0 & | & 0 \\ 0 & -\sin 30 & 0 & 0 & 1 & 0 & | & 0 \\ -1 & 0 & \cos 60 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & -\sin 60 & 0 & 0 & 1 & | & 0 \\ 0 & 0 & \cos 30 & -\cos 60 & 0 & 0 & | & 0 \\ 0 & 0 & \sin 30 & \sin 60 & 0 & 0 & | & 1000 \end{bmatrix} \rightarrow \text{Pivot Row}$$

new $k_{22} = -\frac{1}{2}$ $k_{23} = \frac{\sqrt{3}}{2}$ $k_{24} = 0$ $k_{25} = \frac{\sqrt{3}}{2}$ $k_{26} = \frac{1}{2}$ $P_1 = 0$
 $k_{23} = 0$ $k_{33} = \frac{1}{2}$ $k_{43} = -\frac{\sqrt{3}}{2}$ $k_{53} = -\frac{1}{2}$ $k_{63} = \frac{\sqrt{3}}{2}$ $P_2 = 0$
 $k_{24} = 0$ $k_{34} = 1$ $k_{44} = 0$ $k_{54} = 0$ $k_{64} = 0$ $P_3 = 0$
 $k_{25} = 1$ $k_{35} = 0$ $k_{45} = 0$ $k_{55} = 0$ $k_{65} = 0$ $P_4 = 0$
 $k_{26} = 0$ $k_{36} = 0$ $k_{46} = 1$ $k_{56} = 0$ $k_{66} = 0$ $P_5 = 1000$

$$\begin{bmatrix} 1 & -\frac{\sqrt{3}}{2} & 0 & 1 & 0 & 0 & | & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 1 & 0 & | & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 1 & 0 & 0 & | & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{2} & 0 & 0 & 1 & | & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & 0 & | & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & | & 1000 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{\sqrt{3}}{2} & 1 & -\frac{\sqrt{3}}{2} & 0 & 1 & 0 & 0 & | & 0 \\ 0 & 1 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & | & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 1 & 0 & | & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{2} & 0 & 0 & 1 & 0 & | & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & 0 & | & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & | & 1000 \end{bmatrix} \rightarrow \text{Pivot Row}$$

Interchange and Normalize

new $k_{12} = -\frac{1}{2}$ $k_{22} = \frac{\sqrt{3}}{2}$ $k_{42} = -\frac{\sqrt{3}}{2}$ $k_{52} = 0$ $k_{62} = \frac{1}{2}$ $P_1 = 0$
 $k_{14} = 0$ $k_{24} = \frac{\sqrt{3}}{2}$ $k_{44} = 0$ $k_{54} = 1$ $k_{64} = \frac{\sqrt{3}}{2}$ $P_2 = 0$
 $k_{15} = 0$ $k_{25} = 1$ $k_{45} = 0$ $k_{55} = 0$ $k_{65} = 0$ $P_3 = 0$
 $k_{16} = 0$ $k_{26} = 0$ $k_{46} = 1$ $k_{56} = 0$ $k_{66} = 0$ $P_4 = 0$
 $P_5 = 1000$

$$\begin{array}{c} \curvearrowright \end{array} \left[\begin{array}{cccccc|c} 1 & 0 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\sqrt{3}/2 & -2\sqrt{3}/2 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{3}/6 & -\sqrt{3}/3 & 1 & 0 & 0 \\ 0 & 0 & -\sqrt{3}/2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2\sqrt{3}/3 & \sqrt{3}/3 & 0 & 0 & 1000 \end{array} \right] \rightarrow \begin{array}{c} \begin{array}{c} -1/2 \\ -\sqrt{3}/2 \\ -\sqrt{3}/2 \\ 0 \\ -\sqrt{3}/6 \end{array} \left[\begin{array}{cccccc|c} 1 & 0 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\sqrt{3}/2 & -2\sqrt{3}/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 0 & 866.0254038 \\ 0 & 0 & -\sqrt{3}/2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{3}/6 & \sqrt{3}/3 & 1 & 0 & 0 \end{array} \right] \rightarrow \text{Pivot Row}
 \end{array}$$

Interchange and normalize

$k_{14} = 1/4$ $k_{24} = -\sqrt{3}/2$ $k_{44} = \sqrt{3}/4$ $k_{54} = 1$ $k_{64} = -\sqrt{3}/4$ $P_1 = 433.0127019$
 $k_{15} = 0$ $k_{25} = 0$ $k_{45} = 0$ $k_{55} = 0$ $k_{65} = 1$ $P_2 = 500$
 $k_{16} = 0$ $k_{26} = 0$ $k_{46} = 1$ $k_{56} = 0$ $k_{66} = 0$ $P_4 = 750$
 $P_5 = 0$
 $P_6 = 250$

$$\begin{array}{c} \curvearrowright \end{array} \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 1/4 & 0 & 0 & 433.0127019 \\ 0 & 1 & 0 & -\sqrt{3}/2 & 0 & 0 & 500 \\ 0 & 0 & 1 & 1/2 & 0 & 0 & 866.0254038 \\ 0 & 0 & 0 & \sqrt{3}/4 & 0 & 1 & 750 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{3}/4 & 1 & 0 & 250 \end{array} \right] \rightarrow \begin{array}{c} \begin{array}{c} 1/4 \\ -\sqrt{3}/2 \\ 1/2 \\ 0 \\ \sqrt{3}/4 \\ -\sqrt{3}/4 \end{array} \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 1/4 & 0 & 0 & 433.0127019 \\ 0 & 1 & 0 & -\sqrt{3}/2 & 0 & 0 & 500 \\ 0 & 0 & 1 & 1/2 & 0 & 0 & 866.0254038 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3}/4 & 0 & 1 & 750 \\ 0 & 0 & 0 & -\sqrt{3}/4 & 1 & 0 & 250 \end{array} \right] \rightarrow \text{Pivot Row}
 \end{array}$$

Interchange and normalize

$$\begin{array}{llllll}
 \text{New } k_{15} = 0 & k_{25} = 0 & k_{35} = 0 & k_{45} = 0 & k_{65} = 1 & P_1 = 433.0127019 \\
 k_{16} = 0 & k_{26} = 0 & k_{36} = 0 & k_{56} = 1 & k_{66} = 0 & P_2 = 500 \\
 & & & & & P_3 = 866.0254038 \\
 & & & & & P_5 = 750 \\
 & & & & & P_6 = 250
 \end{array}$$

$$\begin{array}{c} \curvearrowright \end{array}
 \left[\begin{array}{cccccc|c}
 1 & 0 & 0 & 0 & 0 & 0 & 433.0127019 \\
 0 & 1 & 0 & 0 & 0 & 0 & 500 \\
 0 & 0 & 1 & 0 & 0 & 0 & 866.0254038 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 750 \\
 0 & 0 & 0 & 0 & 1 & 0 & 250
 \end{array} \right] \rightarrow \left[\begin{array}{cccccc|c}
 1 & 0 & 0 & 0 & 0 & 0 & 433.0127019 \\
 0 & 1 & 0 & 0 & 0 & 0 & 500 \\
 0 & 0 & 1 & 0 & 0 & 0 & 866.0254038 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 250 \\
 0 & 0 & 0 & 0 & 0 & 1 & 750
 \end{array} \right]$$

Interchange and normalize

$$\left[\begin{array}{cccccc}
 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1
 \end{array} \right]
 \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ A_x \\ A_y \\ B_y \end{bmatrix}
 =
 \begin{bmatrix} 433.0127019 \\ 500 \\ 866.0254038 \\ 0 \\ 250 \\ 750 \end{bmatrix}$$

Therefore,

$$F_1 = 433.0127 \text{ N (T)}$$

$$F_2 = 500 \text{ N (C)}$$

$$F_3 = 866.0254 \text{ N (C)}$$

$$A_x = 0$$

$$A_y = 250 \text{ N } \uparrow$$

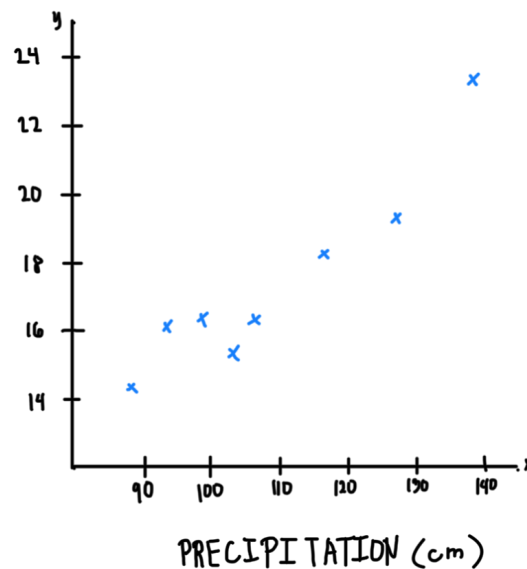
$$B_y = 750 \text{ N } \uparrow$$

3. In water-resources engineering, the sizing of reservoirs depends on accurate estimates of water flow in the river that is being impounded. For some rivers, long-term historical records of such flow data are difficult to obtain. In contrast, meteorological data on precipitation is often available for many years past. Therefore, it is often useful to determine a relationship between flow and precipitation. This relationship can then be used to estimate flows for years when only precipitation measurements were made. The following data are available for a river that is to be dammed:

Precipitation (cm)	88.9	108.5	104.1	139.7	127	94	116.8	99.1
Flow (m^3/s)	14.6	16.7	15.3	23.2	19.2	16.1	18.1	16.6

- Plot the data points with precipitation on the x-axis and flow on the y-axis. (5 points)
- Develop a linear model that relates precipitation (independent variable) to flow (dependent variable) and discuss the limitations of the model and under what conditions would you expect the model to perform poorly. (10 points)
- Estimate the flow if the precipitation is 120 cm. (5 points)
- Following part (c), estimate what fraction of the precipitation is lost via processes such as evaporation, deep groundwater infiltration, and consumptive use, if the area of the river basin is 980 km^2 . (5 points)

For closer look,



(b) $y = a + bx$

y = independent variable = F

x = independent variable = P

$$n+1 = 8$$

$$n = 7$$

$$y = a + bx$$

$$F = a + bP$$

$$a = \bar{F} - b\bar{P}$$

$$b = \frac{\sum P_i (P_i - \bar{P})}{\sum P_i (P_i - \bar{P})}$$

$$\bar{F} = \frac{\sum F_i}{n+1} = \frac{14.6 + 16.7 + 15.3 + 23.2 + 19.2 + 16.1 + 18.1 + 16.6}{8} = 17.475$$

$$\bar{P} = \frac{\sum P_i}{n+1} = \frac{88.9 + 108.5 + 104.1 + 139.7 + 127 + 94 + 116.8 + 99.1}{8} = 109.7625$$

$$b = \frac{14.6(88.9 - \bar{P}) + 16.7(108.5 - \bar{P}) + 15.3(104.1 - \bar{P}) + 23.2(139.7 - \bar{P}) + 19.2(127 - \bar{P}) + 16.1(94 - \bar{P}) + 18.1(116.8 - \bar{P}) + 16.6(99.1 - \bar{P})}{88.9(88.9 - \bar{P}) + 108.5(108.5 - \bar{P}) + 104.1(104.1 - \bar{P}) + 139.7(139.7 - \bar{P}) + 127(127 - \bar{P}) + 94(94 - \bar{P}) + 116.8(116.8 - \bar{P}) + 99.1(99.1 - \bar{P})}$$

$$b = \frac{309.8025}{2073.95875} = 0.1499773683$$

$$a = \bar{F} - b\bar{P}$$

$$= 17.475 - 0.1499773683(109.7625)$$

$$= 1.078966614$$

$$F = 1.079 + 0.1499P$$

③ $P = 120 \text{ cm}$

$$F = 1.079 + 0.1499P$$

$$F = 1.079 + 0.1499(120)$$

$$F = 19.007 \text{ m}^3/\text{s}$$

d. $A = 980 \text{ km}^2 \left(\frac{1000 \text{ m}}{1 \text{ km}} \right)^2 = 980 (10)^6 \text{ m}^2$

Total volume of precipitation:

$$\begin{aligned} V_{TP} &= AP \\ &= 980 (10)^6 (1.2) \\ &= 1.176 (10)^9 \text{ m}^3 \end{aligned}$$

Annual runoff volume (From flow):

$F = 19.007 \text{ m}^3/\text{s}$ means the river discharges that amount continuously over a year, so

$$\begin{aligned} V_{\text{runoff}} &= F (\text{seconds in a year}) \\ &= 19.007 [365 \text{ days} \left(\frac{24 \text{ hrs}}{1 \text{ day}} \right) \left(\frac{3600 \text{ s}}{1 \text{ hr}} \right)] \\ &= 599.4048 (10)^6 \text{ m}^3 \end{aligned}$$

Fraction of precipitation that becomes runoff:

$$f_{\text{runoff}} = \frac{V_{\text{runoff}}}{V_{TP}} = \frac{599.4048 (10)^6 \text{ m}^3}{1.176 (10)^9 \text{ m}^3} = 0.5097$$

Fraction lost to other processes,

$$\begin{aligned} f_{\text{lost}} &= 1 - f_{\text{runoff}} \\ &= 1 - 0.5097 \\ &= 0.4903 \text{ or } 49.03\% \end{aligned}$$

Therefore, about 49.03% of the precipitation is estimated to be lost through evaporation, deep groundwater infiltration, and consumptive use, and the remaining 50.97% becomes surface runoff that flows in the river.